HE 215 : Nuclear & Particle Physics Course

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August 2018 Lectures

Outline I

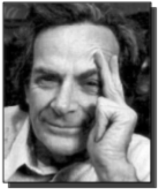
- Quantum Electrodynamics
  - Quantum Electrodynamics (QED)
  - Standard Model Lagrangian
  - Mandelstam Variables
  - Quantum Chromodynamics (QCD)
  - Weak Interactions (leptons)
  - Weak Interactions of Quarks
  - Decays and Conservation Laws

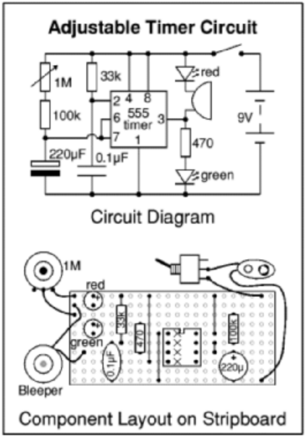
Chapter 2: Elementary Particle Dynamics in Griffiths

Feynman Diagrams

Invented by Richard Feynman to aid calculation of scattering amplitudes, cross sections and decay rates

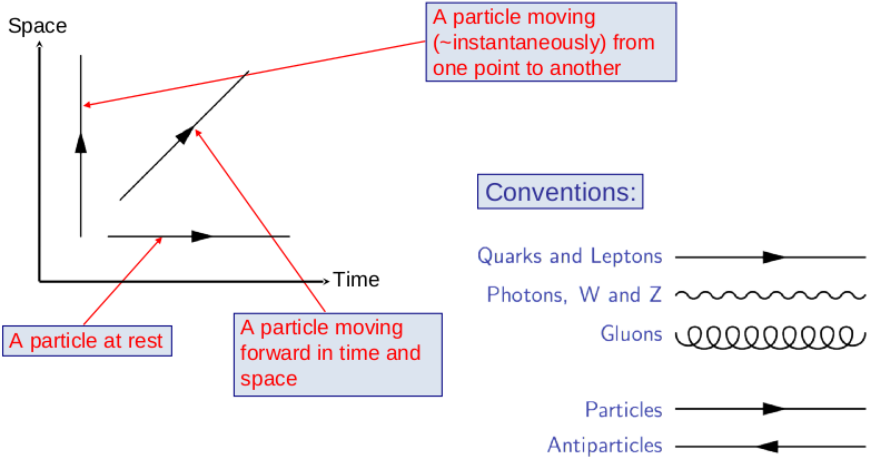
Feynman Diagrams are like circuit diagrams – they show what is connected to what but not the detailed momentum vectors – lengths and angles are not relevant





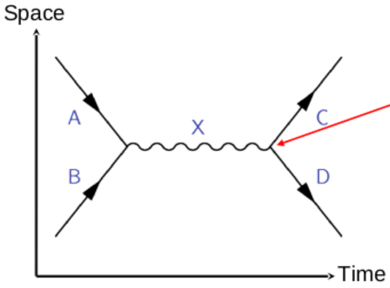
http://www.kpsec.freeuk.com/cdiags.htm

Basics

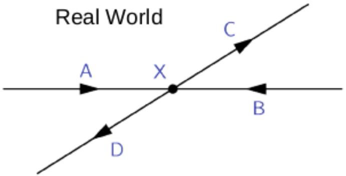


Space
Time
A particle moving (~instantaneously) from one point to another
A particle at rest
A particle moving forward in time and space
Conventions:
Quarks and Leptons
Photons, W and Z
Gluons
Particles
Antiparticles

Annihilation/Formation Diagram. Particles A and B collide to form particle X which later decays to C and D



At each vertex, electric charge must be conserved and, except in Weak Interactions, quark or lepton flavours



Feynman Diagrams

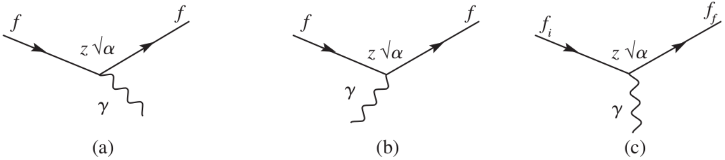
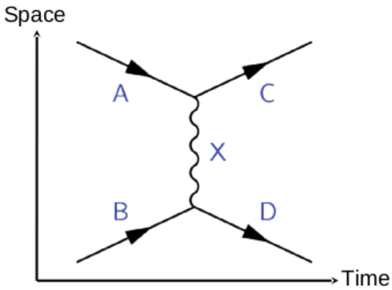


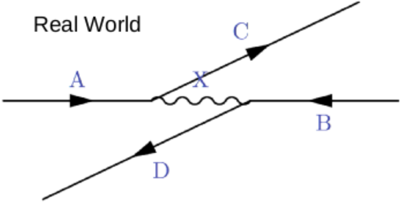
Fig. 5.16. The electromagnetic vertex.

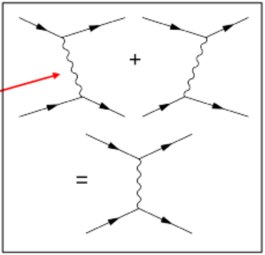
An important element of the diagrams is the vertex, shown in Fig. 5.16 for the electromagnetic interaction. The particles f are fermions, of the same type on the two sides of the vertex, of electric charge z. In Fig. 5.16(a) the initial f disappears in the vertex, while two particles appear in the final state: a fermion f and a photon. The initial state in Fig. 5.16(b) contains a fermion f and a photon that disappear at the vertex; in the final state there is only one fermion f. The two cases represent the emission and the absorption of a photon. Actually the mathematical expression of the two diagrams is the same, evaluated at different values of the kinematic variables, namely the four-momenta of the photon. Therefore, we can draw the diagram in a neutral manner, as in Fig. 5.16(c) (where we have explicitly written the indices i and f for 'initial' and 'final').

Feynman Diagrams

Exchange Diagram. Particles A scatters off particle B by exchanging particle X. Particle A becomes particle C and B becomes D





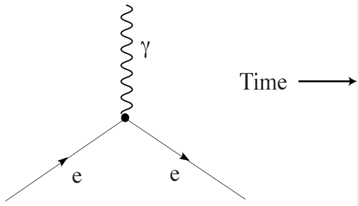


We don't know if A emitted X and B absorbed it or vice versa

Quantum Electrodynamics (QED)

Primitive (Very Basic) Vertex of QED

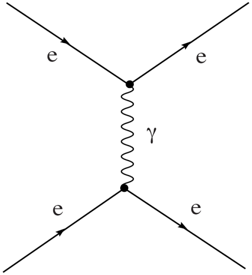
All EM phenomena are ultimately reducible to the following elementary process:



It is just a qualitative description of an interaction (a kind of spacetime diagram).

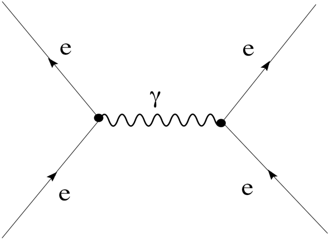
NOTE: time positive left-to-right

From this primitive vertex we can build a Feynman diagram describing the scattering of one electron from another (Møller scattering)
$e^- + e^- \rightarrow e^- + e^-$



You may have called this Coulomb repulsion. Now we will say that the electron-electron interaction is mediated by the exchange of a photon.

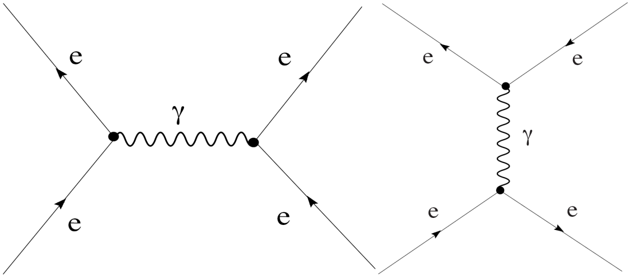
We can rotate this diagram 90 degrees and get: Now we have positrons! And annihilation. This is Bhabha scattering.
$e^- + e^+ \rightarrow e^- + e^+$



QED - Feynman Diagrams

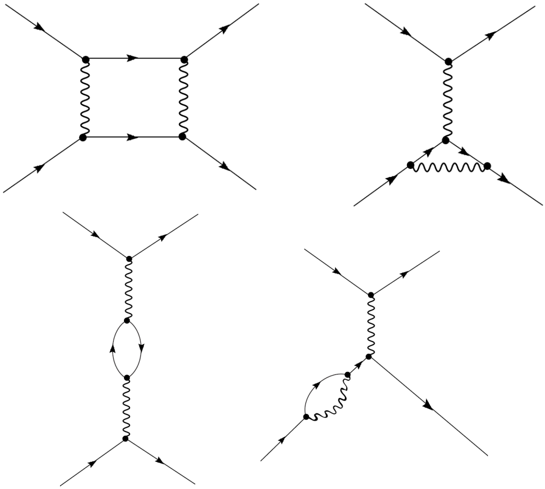
In fact, both of the below are Bhabha scattering:

$$e^- + e^+ \rightarrow e^- + e^+$$



Notice that these two very different looking diagrams have the same initial and final states. These initial and final particles are observable and represented by external lines. The photon is virtual and represented by an internal line. (We will later call these “s-channel” and “t-channel” versions of the same process.)

Are there more ways to draw (for example) Møller scattering (same initial and final states)? Sure, an infinite number of ways. 4-vertex diagrams:



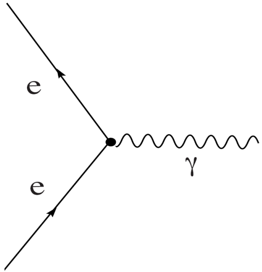
QED - Feynman Diagrams

To represent a physical interaction have to draw an infinite number of squiggly diagrams of ever-increasing complexity. To calculate a cross section need to do an infinite number of integrals. Thanks Feynman!

Each time you add a vertex to a diagram you multiply its contribution by 1/137 (the fine-structure constant). So, just keeping the 2-vertex diagrams often gives a good approximation to the whole process (Leading Order/LO).

QED - Feynman Diagrams

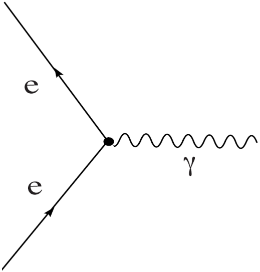
While lengths, angles, etc. on Feynman diagrams are not physically meaningful, the laws of physics are still imposed. Quantum numbers are conserved at each vertex (eg. charge). It is pretty easy to draw a diagram which does not conserve momentum. Take a piece of the Bhabha scattering diagram:



However, once we learn the Feynman rules we will see that this has zero cross section as the rules impose momentum and energy conservation.

Virtual Particles

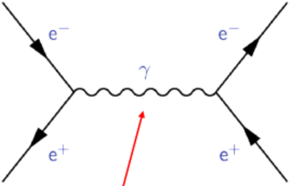
This diagram does not conserve momentum???
Show this in assignment

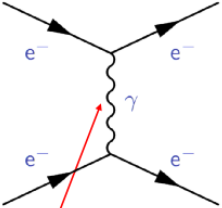


Just remember that the internal lines are virtual particles and so do not have the same mass as their real-world equivalent particles (they can be "off-shell").

Virtual Particles

In both previous cases particle X is 'virtual' and the time it exists is governed by the uncertainty principle $\Delta E\ \Delta t \sim \hbar$. The mass of particle X is usually not its rest mass



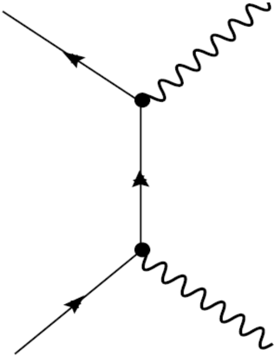


If and electron and positron annihilate, X is a photon (γ) with zero charge, zero momentum and energy $2E_e$ and hence an apparent mass of $2E_e/c^2$

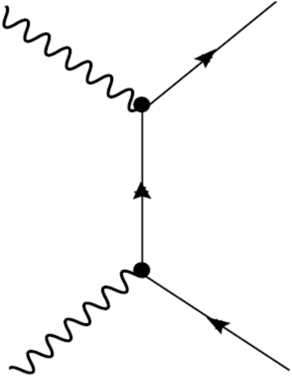
If two electrons scatter, X is a photon (γ) with zero charge, momentum $< 2p_e$ and zero energy and hence an apparent imaginary mass of $< 2\sqrt{-p_e^2}/c^2$

$$E^2 = p^2c^2 + m^2c^4$$

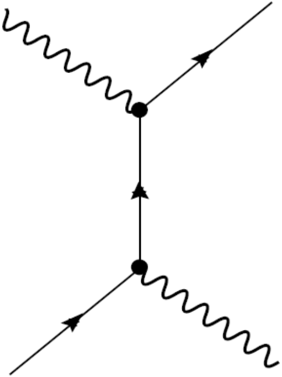
More QED Processes



Pair Annihilation
$e^- + e^+ \to \gamma + \gamma$



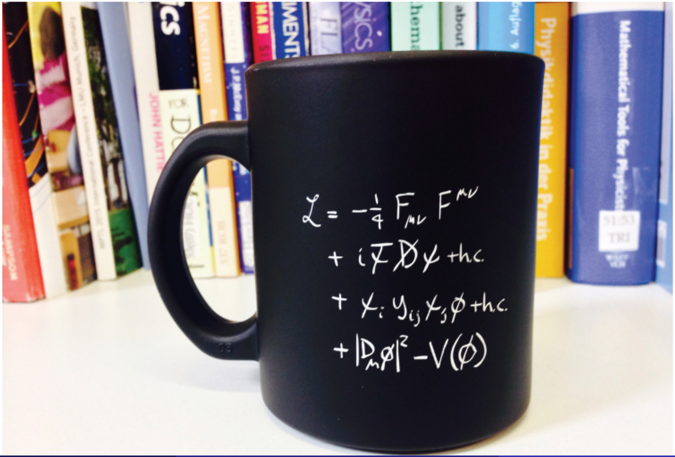
Pair Production
$\gamma + \gamma \to e^- + e^+$



Compton Scattering
$\gamma + e^- \to \gamma + e^-$

Standard Model Lagrangian

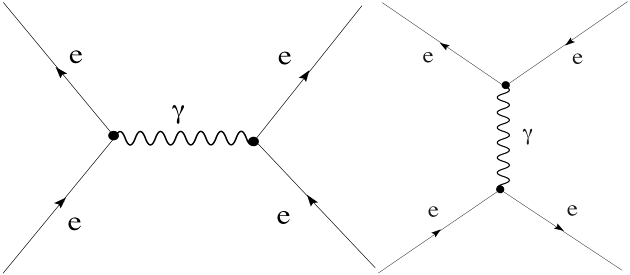
Nice plain english article on Standard Model titled
"Let's have a coffee with the Standard Model of particle physics"
is available at:
`http://iopscience.iop.org/article/10.1088/1361-6552/aa5b25`



Mandelstam Variables

In earlier lecture, we came across Bhabha scattering

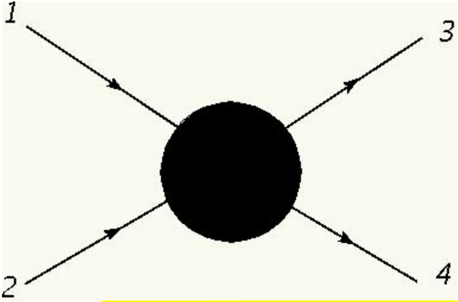
$$e^- + e^+ \rightarrow e^- + e^+$$



The one on the left is called s-channel diagram and the one on the right is called t-channel diagram.
Lets check where this naming comes from!

Mandelstam Variables

Stanley Mandelstam defined some useful kinematic variables for **2-body scattering**



Three invariants:

$$\begin{aligned} s &= (p_1 + p_2)^2 \\ t &= (p_1 - p_3)^2 \\ u &= (p_1 - p_4)^2 \end{aligned}$$

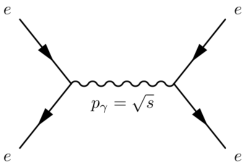
So, an **s-channel** interaction is annihilation and s is the invariant mass^2 of the propagator.

t-channel is scattering - a momentum transfer interaction and t is the square of the momentum transferred.

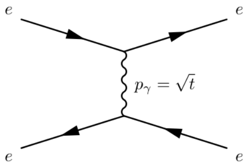
(note: $s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2$)

Mandelstam Variables

$$e^+e^- \to e^+e^-$$

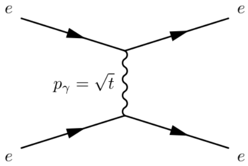


s-channel

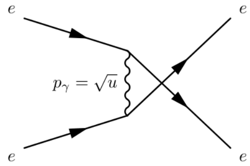


t-channel

$$e^-e^- \to e^-e^-$$



t-channel



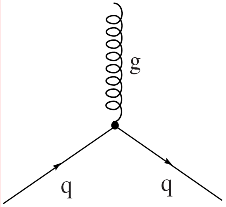
u-channel

Quantum Chromodynamics (QCD)

Quantum Chromodynamics is the theory of the strong interaction. Colour plays the role of charge in QED.

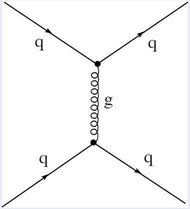
Primitive Vertex of QCD

The fundamental elementary process of QCD:

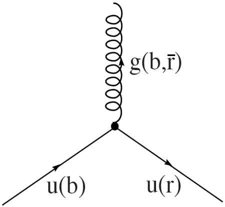


Quantum Chromodynamics (QCD)

Quark-Quark Interactions are Mediated by Gluons



Looks just like QED. However, there is not just one type of charge, there are three colours: red, green, blue.

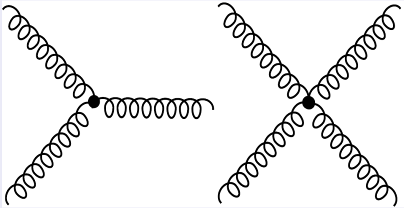


8 different gluons with various colour combinations are possible.

Quantum Chromodynamics (QCD)

The gluons themselves carry colour.

Gluon-Gluon Vertices are Possible



So, we can make "glueballs" whereas "photoballs" are not possible.

Another difference with QED: coupling constant

$$\alpha = \frac{1}{137} \qquad \alpha_s > 1$$

The constant is not constant! At short distance scales QCD exhibits “asymptotic freedom”. Consider charge screening in a di-electric medium:

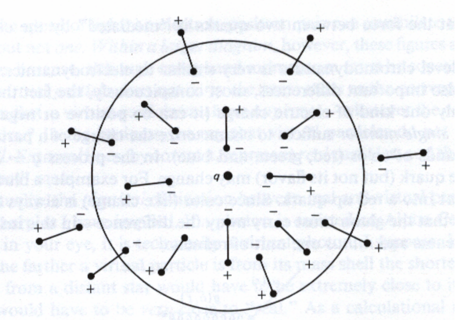
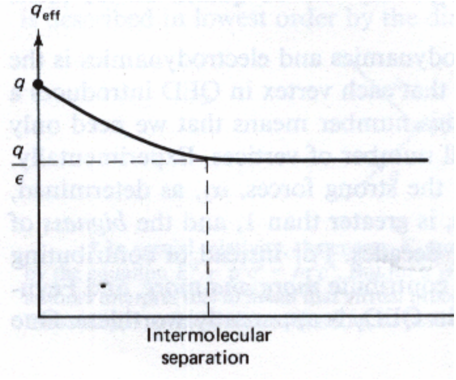
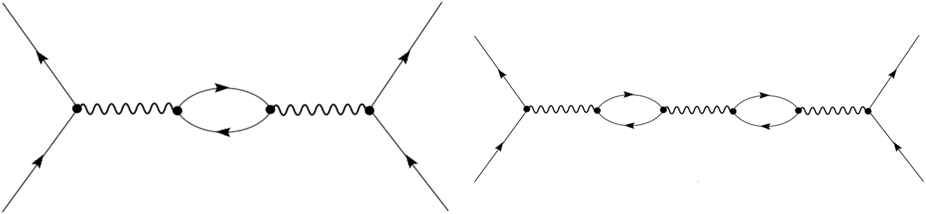


Figure 2.1 Screening of a charge q by a dielectric medium.



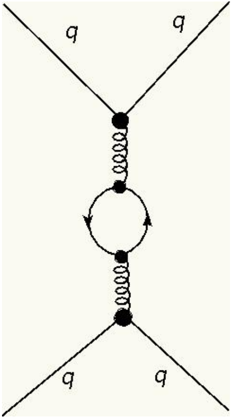
Running Coupling “Constant”

Of course, we don’t have a di-electric medium, just vacuum.

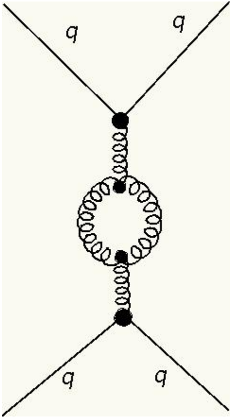


What we call the charge of the electron is really just the fully-screened effective charge.

In QCD something similar happens, but two effects fight (note time bottom to top):



Screening



Anti-Screening

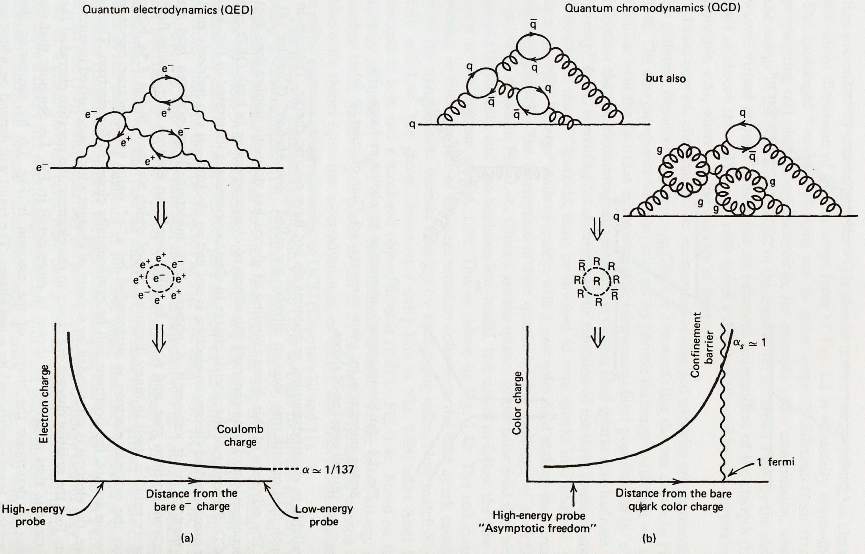
QCD - Running Coupling "Constant"

Anti-screening wins because there are more different types of gluons than there are types of quarks.

$$a \equiv 2f - 11n$$

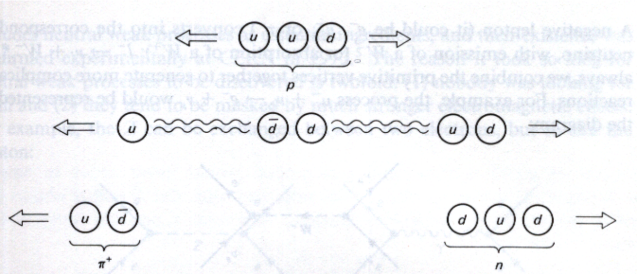
where
f is the number of flavours (6) and
n is the number of colours (3),
$\implies a = -21$.
Negative numbers means anti-screening.

QCD - Running Coupling “Constant”



Taken from Quarks & Leptons – Halzen & Martin

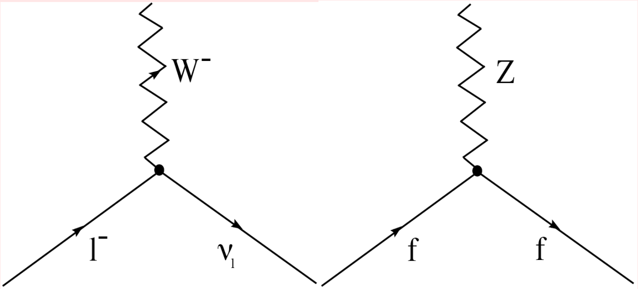
Unlike EM charge, no naturally occuring particles carry colour - confinement.



So, how do we "see" a quark? Looks for jets

All quarks and leptons participate in the weak interaction (no "charge" required). There are charged boson mediators and a neutral mediator.

Fundamental Vertices of the Weak Interaction (leptons)



Note: this is the first time we have seen massive propogators. The emission of the charged mediator converts the charged lepton into a neutrino.

$M_W = 80.385$ GeV & $M_Z = 91.1876$ GeV (Taken from PDG 2017)

Weak Interactions

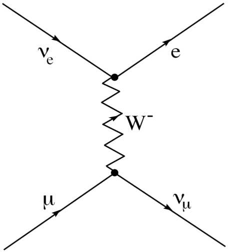
The weak interaction was postulated to explain some decays which behaved in a funny way (eg. slow decay of the neutron). The "weakness" of the force could have been described by a small coupling constant (α_W). Instead consider giving it the same coupling strength as the EM interaction but a heavier field boson mediating the force!

The probability of exchanging a W is small not because it is less likely to be emitted but because it is massive (a short-range force).

$$M_{W,Z} \sim \sqrt{\frac{4\pi\alpha}{G}} \sim 90 GeV$$

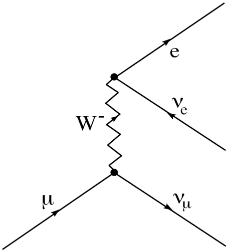
where G had been measured from β decay rates.

Neutrino scattering must proceed via the weak interaction



$\mu^- + \nu_e \to \nu_\mu + e^-$

muon decay as well

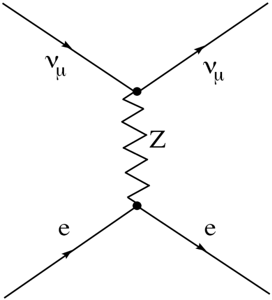


$\mu^- \to \nu_\mu + e^- + \bar{\nu}_e$

Weak Interactions (leptons)

We can also have neutrino scattering with a Z

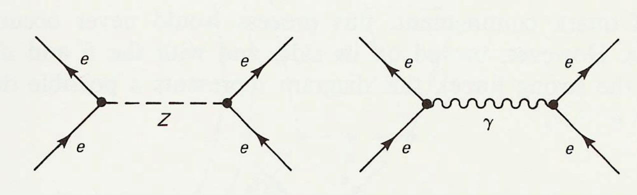
$$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$$



Weak Interactions (leptons)

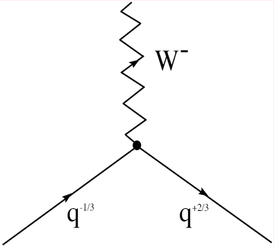
We'll see later that EM and weak interactions are unified in electroweak theory. The Z and the γ have a lot in common

$$e^- + e^+ \rightarrow e^- + e^+$$



Weak Interactions of Quarks

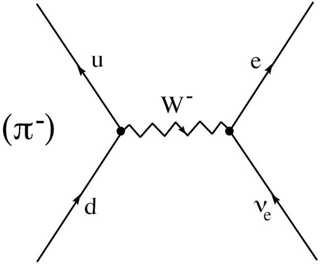
Fundamental Weak Interaction of Quarks



While the weak interaction conserves electron number, muon number, etc. it does not preserve quark "numbers" (ie. there are not the same number of up quarks after a weak interaction as there were before it).
The outgoing quark has the same colour as the incoming quark but a different flavour.
This is a Flavour-Changing Charged Current (FCCC).

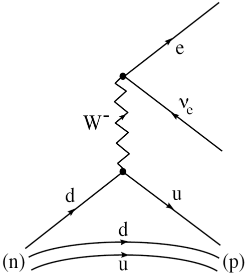
Weak Interactions of Quarks

The conversion of one flavour of quark to another allows for important types of decays in nature



Pion Decay

$\pi^- \rightarrow e^- + \bar{\nu}_e$



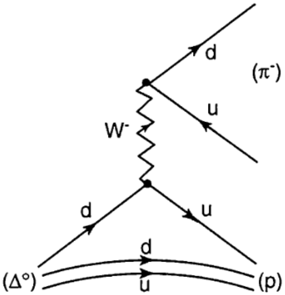
Neutron Decay

$n \rightarrow p + e^- + \bar{\nu}_e$

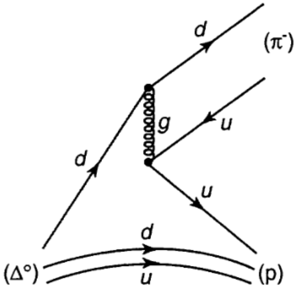
There are also quark interactions mediated by Z but they are not flavour-changing
(ie. there are no flavour-changing neutral currents (FCNC) in the SM).

Weak Interactions of Quarks

$\Delta^0(udd)$ Weak Decay
$\Delta^0 \to p + \pi^-$



$\Delta^0(udd)$ Strong Decay
$\Delta^0 \to p + \pi^-$



Δ^0 (udd) decays via both Strong & Weak interaction but weak meachanism has an immeasurably small contribution.
How about Δ^{++} (uuu) ? Can you guess its decay modes?

Weak Interactions of Quarks

We have been changing flavour within a generation, but what about between generations? The weak force does not couple to:

$$\begin{pmatrix} u \\ d \end{pmatrix}, \quad \begin{pmatrix} c \\ s \end{pmatrix}, \quad \begin{pmatrix} t \\ b \end{pmatrix}.$$

Instead, it couples to

$$\begin{pmatrix} u \\ d' \end{pmatrix}, \quad \begin{pmatrix} c \\ s' \end{pmatrix}, \quad \begin{pmatrix} t \\ b' \end{pmatrix}$$

Where d', s', b' are linear combinations of the corresponding physical quarks.

Weak Interactions of Quarks - CKM

The mixing of quark generations is expressed in the **Cabibbo-Kobayashi-Maskawa (CKM) matrix**

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{\mathrm{ud}} & V_{\mathrm{us}} & V_{\mathrm{ub}} \\ V_{\mathrm{cd}} & V_{\mathrm{cs}} & V_{\mathrm{cb}} \\ V_{\mathrm{td}} & V_{\mathrm{ts}} & V_{\mathrm{tb}} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\begin{pmatrix} 0.97383 & 0.2272 & 0.00396 \\ 0.2271 & 0.97296 & 0.04221 \\ 0.00814 & 0.04161 & > 0.68 \end{pmatrix}$$

This is not the unit matrix. There is no quark equivalent of “electron number” OR “muon number” OR “tau number”.

Interactions Summary

- EM interactions are mediated by the photon. Coupling is to charge with strength α
- Strong interactions are mediated by gluons. Coupling is to colour with strength α_s.
- Weak interactions are mediated by W+/- and Z. W can change quark flavour. Coupling is the same as EM but the gauge bosons are heavy.
- If a process can occur by several different mechanisms, it proceeds more often (and more quickly) by the strongest available interaction.

Decays and Conservation Laws

Every particle decays into lighter particles unless prevented from doing so by some conservation law

The list of stable particles is very short. Most particles we produce decay and do so by several different mechanisms (must measure branching ratios). Decays via different interactions are characterized by very different lifetimes:

- strong decay $\sim 10^{-23}s$
- EM decay $\sim 10^{-16}s$
- Weak decay $\sim 10^{-13}s$ - 15 minutes

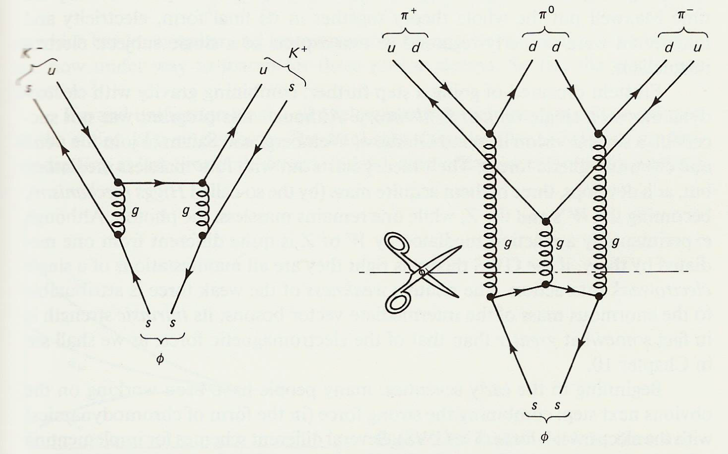
A large mass difference also allows the decay to proceed quickly.

Conservation Laws

Of course, any decay must conserve energy and momentum. However, there are many other (non-kinematic) conservation laws which dictate what decays will and will not occur. These are conserved at each vertex in a Feynman Diagram and in the interaction as a whole:

1. Electric charge
2. Colour. EM and weak interactions do not affect colour. Gluons can “carry away” colour but it is still conserved overall.
3. Baryon number. The total number of quarks is constant. We don’t see quarks directly though, so we speak in terms of baryons.
4. Electron number, muon number, tau number. Weak and EM forces affect leptons. Weak can change particle but within its generation. (no hadronic equivalent)
5. Approximate conservation of flavour. Only weak interactions violate this.
6. The OZI rule. If a diagram can be cut in two by snipping only gluon lines then it is **suppressed** and the lifetime may be unusually long.

OZI Rule (time up)



Electric Charge

Quantum Numbers are quantised properties of particles that are subject to constraints. They are often related to symmetries

Electric Charge Q is conserved in all interactions

Strong Interaction	$\pi^- + p \rightarrow K^0 + \Lambda^0$ Q: -1 +1 0 0	✓
Weak Interaction	$K^0 \rightarrow \pi^+ + \pi^-$ Q: 0 +1 -1	✓

Baryon Number B

Baryon number is the net number of baryons or the net number of quarks ÷ 3

Baryons have B = +1
Antibaryons have B = -1
Everything else has B = 0

or

Quarks have B = +⅓
Antiquarks have B = -⅓
Everything else has B = 0

Baryons = qqq = ⅓ + ⅓ + ⅓ = 1 Mesons = $q\bar{q}$ = ⅓ + (-⅓) = 0

Baryon Number B is conserved in Strong, EM and Weak interactions

Total (quarks – antiquarks) is constant

Baryon Number

Strong Interaction	$$\pi^- + p \to K^0 + \Lambda^0$$ B: -0 +1 0 +1	✓
Weak Interaction	$$\Lambda^0 \to p + \pi^-$$ B: +1 +1 0	✓

Since the proton is the lightest baryon it cannot decay if B is conserved e.g:

$$p \nrightarrow e^+ + \pi^0$$

B: +1 0 0 ✗

Lepton Number L

Leptons have L = +1
Antileptons have L = -1
Everything else has L = 0

Lepton Number L is conserved in Strong, EM and Weak interactions but is also separately conserved within lepton families:

e^- and ν_e have $L_e = 1$	e^+ and $\bar{\nu}_e$ have $L_e = -1$
μ^- and ν_μ have $L_\mu = 1$	μ^+ and $\bar{\nu}_\mu$ have $L_\mu = -1$
τ^- and ν_τ have $L_\tau = 1$	τ^+ and $\bar{\nu}_\tau$ have $L_\tau = -1$

L_e, L_μ and L_τ are separately conserved

Lepton Number

Pair Production

$$\gamma + N \rightarrow e^+ + e^- + N$$

	γ	N	e^+	e^-	N
L_e:	0	0	-1	+1	0

✓

Pion Decay

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

	π^+	μ^+	ν_μ
L_μ:	0	-1	+1

✓

Muon Decay

$$\mu^+ \rightarrow e^+ + \nu_e + \overline{\nu}_\mu$$

	μ^+	e^+	ν_e	$\overline{\nu}_\mu$
L_e:	0	-1	+1	0
L_μ:	-1	0	0	-1
L:	-1	-1	+1	-1

✓

Lepton Number

Radiative Decay

$$\mu^+ \nrightarrow e^+ + \gamma$$

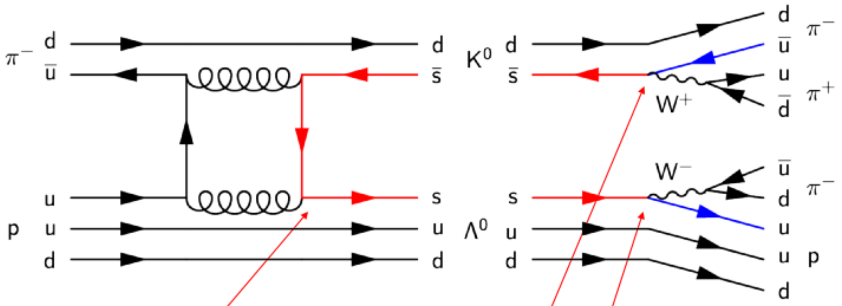
	μ^+	e^+	γ		
L_e:	0	-1	0	Forbidden	✗
L_μ:	-1	0	0	Forbidden	✗
L:	-1	-1	0	OK	✓

L is conserved but neither L_e or L_μ separately

The decay has not been observed and has a "Branching Ratio" $< 10^{-9}$

Strangeness S

Assume pair production of new quark s and antiquark $\bar{s}$ by Strong Interaction but once produced s and $\bar{s}$ can only decay weakly



Strong Production

Weak Decay

Λ^0 is uds	$\overline{\Lambda}^0$ is $\bar{u}\bar{d}\bar{s}$
K^0 is $d\bar{s}$	$\overline{K}^0$ is $\bar{d}s$
K^+ is $u\bar{s}$	K^- is $\bar{u}s$

The K^0 has an antiparticle the $\overline{K}^0$ although it is neutral, unlike the π^0 which is its own antiparticle

Strangeness can be combined with Isospin if

$$Q = I_3 + (B + S)/2$$

Gell Mann – Nishijima relation

The s quark has strangeness S = -1

Strangeness is conserved in Strong and EM Interactions but NOT in Weak Interactions

Likewise charm, bottom, top quantum numbers

Strong and EM Interactions do not change quark flavours.
Number of $(u - \bar{u})$, $(d - \bar{d})$, $(s - \bar{s})$, $(c - \bar{c})$, $(b - \bar{b})$, $(t - \bar{t})$ constant

Weak Interaction changes one quark type to another

Summary

Conserved Quantum Numbers

Quantity		Strong	EM	Weak
Charge	Q	✓	✓	✓
Baryon Number	B	✓	✓	✓
Lepton Number	L	✓	✓	✓
Strangeness	S	✓	✓	✗
Isospin	I	✓	✗	✗
	I_3	✓	✓	✗
Parity	P	✓	✓	✗
Charge Conjugation	C	✓	✓	✗

Unification

Physicists have already unified electricity and magnetism into EM and EM+Weak into electroweak. It is natural to want to unify electroweak and QCD. The running coupling constants hold promise: